Multiple Phase Flow

Burhan S Abdulrazak, Ph.D.
Chemical Engineering Department

Lecture Notes: Separated Flow Model

Introduction

The **separated flow model** is a multiphase flow model that assumes the phases (e.g., gas and liquid) are distinct and flow separately with individual velocities. Unlike the homogeneous flow model, which treats all phases as a single effective fluid, the separated flow model considers the slip between phases, making it more accurate for systems where phase interaction and separation are significant.

Key Assumptions

Distinct Phases:

- The phases (e.g., gas and liquid) maintain distinct identities and flow separately.

Slip Between Phases:

- The gas and liquid phases can have different velocities, defined by the **slip ratio** (S):

$$S = \frac{u_g}{u_l}$$

Where ug and ul are the gas and liquid velocities, respectively.

Steady-State Flow:

- The model typically assumes that the flow is steady and time-independent.

One-Dimensional Flow:

- The analysis is conducted along the pipe's axial direction.

Governing Equations

The separated flow model uses conservation equations for each phase, which are coupled through interphase interactions. These include:

1. Mass Conservation

For each phase (k = g for gas, l for liquid):

$$\frac{\partial}{\partial t}(\alpha_k \rho_k) + \nabla \cdot (\alpha_k \rho_k \mathbf{u}_k) = 0$$

Where:

- αk: Volume fraction of phase k
- ρk: Density of phase k
- uk: Velocity of phase k

2. Momentum Conservation

For each phase:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k \mathbf{u}_k) + \nabla \cdot (\alpha_k \rho_k \mathbf{u}_k \mathbf{u}_k) = \alpha_k \mathbf{F}_k - \nabla p$$

Where:

- Fk: Force acting on phase k (gravity, friction, interphase drag)

3. Energy Conservation

For each phase:

$$\frac{\partial}{\partial t}(\alpha_k \rho_k e_k) + \nabla \cdot (\alpha_k \rho_k e_k \mathbf{u}_k) = \alpha_k q_k + \mathbf{w}_k$$

Where:

- ek: Specific energy of phase k
- qk: Heat transfer
- wk: Work done on the phase

Key Parameters

Slip Ratio (S):

- Measures the velocity difference between phases.
- A higher slip ratio indicates significant phase separation.

Volume Fraction (αk):

- The fraction of the pipe's volume occupied by each phase.

Flow Quality (x):

- The mass fraction of the gas phase in the total flow:

$$x = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_l}$$

4- Mass Flux (G):

- The total mass flow rate per unit area:

$$G = \frac{\dot{m}_g + \dot{m}_l}{A}$$

5- Void Fraction (αg):

- The fraction of the cross-sectional area occupied by the gas phase.

Applications

Oil and Gas Pipelines:

- Predicts phase separation and slip in multiphase pipelines.

Nuclear Reactor Coolant Systems:

- Models gas-liquid flows in cooling channels of boiling water reactors.

Heat Exchangers:

- Analyzes phase behavior in condensers and evaporators.

Chemical Process Engineering:

- Used in distillation columns and two-phase separators.

Advantages

Realistic Assumptions:

- Accounts for phase slip, improving accuracy in systems with significant phase separation.

Broad Applicability:

- Can model a wide range of flow regimes (e.g., stratified, annular).

Enhanced Prediction:

- Provides more detailed insights into flow properties (e.g., pressure drop, phase velocities).

Limitations

Complexity:

- Requires solving separate equations for each phase, increasing computational effort.

Empirical Correlations:

- Often relies on experimental data for slip ratio and interphase forces.

Limited to Two Phases:

- Extending the model to three or more phases adds significant complexity.

Example Calculation: Separated Flow Model

Problem: A gas-liquid mixture flows through a horizontal pipe with the following properties:

- Gas volume fraction (αg) = 0.4
- Gas density (ρg) = 2 kg/m^3
- Liquid density (ρl) = 1000 kg/m^3
- Slip ratio ($S = \frac{u_g}{u_l}$) = 2
- Total mass flow rate ($\dot{m}$) = 50 kg/s
- Pipe diameter (D=0.1 m).

Find:

Mixture density (ρ_m).

Mixture velocity (u_m).

Solution

Mixture Density: The mixture density is given by:

$$\rho_m = \alpha_g \rho_g + (1 - \alpha_g)\rho_l$$

Substituting the values:

$$\rho_m = (0.4)(2) + (1 - 0.4)(1000)$$

$$\rho_m = 0.8 + 600 = 600.8\,\text{kg/m}^3$$

Mixture Velocity: The slip velocity relationship allows us to determine the individual phase velocities:

$$u_g = S \cdot u_l$$

The total mass flow rate is:

$$\dot{m} = \rho_g u_g A_g + \rho_l u_l A_l$$

Where:

- $A_g = \alpha_g A$
- $A_l = (1 - \alpha_g)\, A$

Substitute the known values and solve for u_l and u_g:

- $A = \frac{\pi D^2}{4} = 0.00785\, m^2$

Simplify to get $u_m = \frac{\dot{m}}{\rho_m A}$.

Conclusion

The **separated flow model** is a powerful tool for analyzing multiphase flows with significant phase separation. By incorporating slip between phases, it provides a more realistic representation of real-world systems compared to simpler models like the homogeneous flow model. However, its complexity and reliance on empirical correlations necessitate careful application.